Density distribution for a weakly non-ideal non-uniform plasma

2010 J. Phys. A: Math. Theor. 43 125501

(http://iopscience.iop.org/1751-8121/43/12/125501)

J. Phys. A: Math. Theor. **43** (2010) 125501 (9pp) doi:10.1088/1751-8113/43/12/125501

Density distribution for a weakly non-ideal non-uniform plasma

Anirban Bose and M S Janaki

Saha Institute of Nuclear Physics, I/AF Bidhan Nagar, Kolkata 700 064, India

E-mail: ms.janaki@saha.ac.in

Received 14 August 2009, in final form 10 February 2010
Published 8 March 2010
Online at stacks.iop.org/JPhysA/43/125501

Abstract
An attempt is made to develop an equilibrium kinetic equation for a weakly non-ideal non-uniform plasma utilizing the Bogoliubov–Born–Green–Kirkwood–Yvon hierarchy of equations. The time-independent pair correlation function is shown to be a product of two single-particle non-uniform distribution functions and the binary interaction potential that is taken to be Coulombian. In order to obtain a closed form of kinetic equation, it is necessary to express the first-order corrections to the Vlasov equation arising out of correlations in terms of average plasma potential. The singular nature of the Coulomb potential gives rise to certain divergences that can be removed by the choice of Landau and Debye lengths as the lower and upper limits of the impact parameter. This procedure enables a representation of pairwise interaction potential in terms of average macroscopic potential. The first-order kinetic equation is utilized to obtain a modified Boltzmann distribution that includes the effects of correlations.

PACS numbers: 52.25.Dg, 51.10, +y, 95.30.Qd, 96.25.St

The thermodynamic properties of a plasma can be characterized by defining the coupling parameter Γ which is the ratio of the mean Coulomb potential energy to the mean thermal energy:

$$\Gamma = \frac{e^2}{ak_BT}$$

where $a = n^{-1/3}$ is the order of the average interparticle distance.

Depending on the value of the coupling parameter, three different regimes can be distinguished where the nature of interactions are different. (a) $\Gamma \ll 1$: here, the Debye screening length is much greater than the average interparticle distance and the plasma particles are assumed to be uncorrelated. The plasma behaviour resembling that of an ideal gas can be studied using the Vlasov equation or the associated fluid models. (b) $\Gamma \approx 0.1$: the mean Coulomb energy is not much smaller than the average thermal energy so that correlations between the plasma particles are non-negligible. It is not possible to separate individual and collective degrees of freedom. The plasma is defined as a weakly non-ideal plasma. The theoretical description of such a system falls within the framework of kinetic theory using a perturbation expansion about the plasma parameter $g = 1/n\lambda_D^3 = \Gamma^{3/2}$, where $\lambda_D^2 = \epsilon_0 K_B T/ne^2$ (c) $\Gamma > 1$. In this regime, characterized by a high density or low temperature, the plasma is called a non-ideal plasma. Here particle–particle correlations are important. Such plasmas are difficult to treat theoretically because standard approximations and expansion methods break down.

In the solar core and other astrophysical systems such as brown dwarfs, Jupiter core and stellar atmospheres, $\Gamma \approx 0.1$. Such systems can be considered as weakly non-ideal plasma systems. The advent of femtosecond laser pulses also stimulates the interest of weakly coupled plasmas. The properties of such plasmas cannot be described by the model of an ideal plasma system. From the theoretical point of view, a weakly coupled plasma can be regarded as a test bed for the study of the onset of non-ideality effects [1]. Studies on weakly non-ideal plasmas help in the understanding of how stationary equilibrium is reached within a plasma.

The Vlasov equation is obtained in the mean field limit where each particle interacts with an average field produced by all the other plasma particles, with the particle discreteness effects completely neglected. The collisional dynamics of a homogeneous system with weak long-range interaction is described by the Landau equation [2] that can be derived from the Boltzmann or Fokker–Planck equations. The time evolution of a spatially uniform plasma including the physics of collisions between two shielded particles is described by the Balescu–Lenard [3] equation under the assumption that three-particle correlations are negligible. A general kinetic equation including a Fokker–Planck collisional integral enabling the study of the role of collisions on collective processes was proposed by Lenard and Bernstein [4] under the conditions that the equation preserves conservation of number of particles and yields an equilibrium that is Maxwellian. The Lenard–Bernstein collisional operator was also used by Zakharov and Karpmann [5] to study the collisional damping of nonlinear plasma waves. Analytical [6] and experimental works [7], as well as numerical simulations [8], have studied the slow decay of nonlinear trapping oscillations [9] due to the effects of weak collisional dissipation and also the regime of strong collisionality where no nonlinear plasma waves can be produced. The importance of two-particle correlations in the evolution of a system towards equilibrium is studied through particle-in-cell simulations on nonlinear Landau damping and the results compared with those of toy model simulations [10].

Another method to study the role of collisions on plasma phenomena is through the Bogoliubov–Born–Green–Kirkwood–Yvon (BBGKY) hierarchy description which at the first order in the plasma parameter g retains the effects of pair interactions. For long wavelength electrostatic plasma oscillations, the correlational damping decrement obtained from the linearized BBGKY hierarchy equations was found [11] to greatly exceed the Landau damping decrement. Both the Landau and Balescu–Lenard equations are first-order equations in the parameter g and are known to have a large domain of validity [12] with applicability to a large class of potentials of interaction in various of dimensions of space as well as for multicomponent systems. The equations are extensively used to describe the influence of pair interactions on the time evolution of the plasma and its dissipative characteristics. It would be interesting to examine the effects of pair correlations on the equilibrium properties of the plasma such as nonlinear stationary solutions of Vlasov–Poisson's equation. In one dimension, a stationary solution of the electrostatic Vlasov equation can be constructed as a function of the energy of a single particle. Such solutions are of fundamental importance to plasma physics because together with Poisson's equation, they self-consistently describe a class of nonlinear structures known as Bernstein–Greene–Kruskal (BGK) modes [13].

For an inhomogeneous unmagnetized plasma, in the presence of a purely electrostatic interaction, the density distribution determines the nature of the potential distribution through Poisson's equation. The density profile is obtained through a velocity space integration of the single-particle distribution function that is a solution of the collisionless Vlasov equation. In the case of a more general equilibrium kinetic equation such as the BBGKY hierarchy equation at first order, the nature of the single-particle distribution as well as the density profile should be influenced by the nature of the binary interaction between particles. A straightforward way to understand this influence would be to develop an equilibrium kinetic equation for a pair correlated non-uniform plasma by considering the first two equations of the BBGKY hierarchy. The BBGKY hierarchy at the first order contains the two-particle distribution function that can be expressed in terms of the pair correlation function utilizing the Mayer cluster expansion. The equilibrium second hierarchy equation with the three-particle correlation terms neglected is solved for the spatial dependence of the pair correlation function. For a plasma in equilibrium, the pair correlation function is shown to be a product of two single-particle non-uniform distribution functions and the interaction potential between two particles that is taken to be Coulombian. The pair correlation function is utilized in the first-order equilibrium kinetic equation for electrons and ions to obtain their respective distribution functions in terms of the electrostatic potential in the presence of electron–electron, electron–ion and ion–ion correlation effects.

The $1/r$ nature of the Coulomb interaction between charged particles gives rise to different diverging integrals in the context of the study of correlation effects. The long-range nature of the Coulomb potential gives rise to one kind of divergence, avoided by cutting the impact parameter at λ_D at the upper limit. Such a cutoff matches perfectly with the characteristics of the plasma as the Coulomb potential is modified by the screened Debye potential with its effective range limited up to the Debye length.

At short distances the unbound nature of the potential invalidates the expansion technique of the distribution function with $g = 1/n\lambda_D^3$ as the expansion parameter. Different techniques have been employed to lift this difficulty. For a charged particle system, Bogoliubov [14] suggested a modified Coulomb potential of the following form:

$$\phi = \frac{e^2}{r}[1 - \exp(-\overline{\lambda} r)],$$

that effectively lifts the divergence at small distances. Another procedure [15] comes with the cutting off of the k-space integral for large k which avoids close encounters between the particles. In the non-relativistic quantum mechanical approach [16] the de Broglie wavelength of the charged particles is taken as the finite dimension of the particle. So the Coulomb potential is restricted down to de Broglie wavelength at the lower limit and that eventually puts away the divergence. The problem of divergences has also been tackled by defining various forms of convergent collision integrals [17].

The aim of this paper is to construct the equilibrium density distribution function of plasma particles in the presence of weak correlations in a non-uniform plasma by lifting the divergences in a physically meaningful way.

1. Pair correlation function in an equilibrium non-uniform plasma

For a spatially non-uniform plasma, the equilibrium pair correlation function can be obtained by considering the first two members of the BBGKY hierarchy of equations and neglecting three-particle correlations. Utilizing the Mayer cluster expansion for the two-particle distribution function

$$f_2(\mathbf{X}_1, \mathbf{X}_2) = f_1(\mathbf{X}_1) f_1(\mathbf{X}_2) + g_{12}(\mathbf{X}_1, \mathbf{X}_2),$$

one of the time-independent solutions of the first two members of the BBGKY hierarchy set of equations can be obtained [18] as

$$g_{12}(\mathbf{X}_1, \mathbf{X}_2) = f_1(\mathbf{X}_1) f_1(\mathbf{X}_2) \chi_{12}(\mathbf{x}_1, \mathbf{x}_2), \tag{1}$$

where the single-particle distribution functions $f_1(\mathbf{X}_1)$ and $f_1(\mathbf{X}_2)$ are functions of both position and velocity and χ_{12} describes the positional correlation between particles. Due to the central nature of the Coulomb force, we can write

$$\chi_{12}(\mathbf{x}_1, \mathbf{x}_2) = \chi_{12}(|\mathbf{x}_1 - \mathbf{x}_2|). \tag{2}$$

For a non-uniform plasma in thermal equilibrium, the single-particle distribution function is considered to be of the form

$$f_1(\mathbf{X}_1) = f_M(\mathbf{v}_1) F_1(\mathbf{x}_1), \tag{3}$$

where f_M is a Maxwellian distribution and F_1 denotes the spatially dependent part of the single-particle distribution function. The spatial dependence of the distribution function works out to be the Boltzmann distribution for an equilibrium plasma supporting an electrostatic potential when there are no velocity correlations between particles. Under the approximation $r \ll \lambda_D$, with $r = |\mathbf{x}_1 - \mathbf{x}_2|$ denoting the interparticle distance, the following form [18] is obtained for χ_{12}:

$$\chi_{12}(\mathbf{X}_1, \mathbf{X}_2) = -1 + \exp\left(-\frac{\phi_{12}}{k_B T}\right). \tag{4}$$

While obtaining the above expression it is assumed that if particles 1 and 2 belong to different species, they have attained a common temperature T. This form of pair correlation function satisfies the physical conditions [19] that it tends to zero at very large distances. For $\phi_{12}/k_B T \ll 1$, the expression for χ_{12} can be approximated by

$$\chi_{12}(\mathbf{X}_1, \mathbf{X}_2) = -\frac{\phi_{12}}{k_B T}. \tag{5}$$

The average interaction potential between two particles separated by a distance $|\mathbf{x}_1 - \mathbf{x}_2|$ can be approximated by the Coulomb potential [20]:

$$\phi_{12}(\mathbf{x}_1 - \mathbf{x}_2) = \frac{q^2}{4\pi\epsilon_0 |\mathbf{x}_1 - \mathbf{x}_2|}, \tag{6}$$

to obtain the following pair correlation function:

$$g_{12}(\mathbf{X}_1, \mathbf{X}_2) = \frac{q^2}{4\pi\epsilon_0 k_B T |\mathbf{x}_1 - \mathbf{x}_2|} f_1(\mathbf{X}_1) f_1(\mathbf{X}_2). \tag{7}$$

The form of the pair correlation function given in equation (7) exhibits a singularity [21] at $r = 0$ which arises due to the singular nature of the Coulomb force. However, it is to be noted that this form of g_{12} is obtained under the approximation $\phi_{12}/k_B T \ll 1$ that restricts r to values greater than r_{av}^2/λ_D^2, where r_{av} is the mean interparticle distance.

We note that the expression in equation (7) describes a time-independent solution for the pair correlation function that differs from well-known [15] equilibrium pair correlation functions as it supports non-uniform single-particle distribution functions. A knowledge of the two-particle correlation function is essential for calculating the thermodynamic properties of a plasma in equilibrium [22].

2. Equilibrium kinetic equation for a weakly non-ideal plasma

The kinetic equation in the presence of correlations can be written by making use of equation (7) for the equilibrium two-particle correlation function in the first-order kinetic equation given below:

$$\mathbf{v}_1 \cdot \frac{\partial f_1}{\partial \mathbf{x}_1} - \frac{n_0}{m}\int \mathrm{d}\mathbf{X}_2 f_1(\mathbf{X}_2)\nabla_{\mathbf{x}_1}\phi_{12}\cdot\frac{\partial f_1(\mathbf{X}_1)}{\partial \mathbf{v}_1} = \frac{n_0}{m}\int \mathrm{d}\mathbf{X}_2 \nabla_{\mathbf{x}_1}\phi_{12}\cdot\frac{\partial g_{12}}{\partial \mathbf{v}_1}$$
$$= -\frac{n_0}{mk_BT}\int \mathrm{d}\mathbf{x}_2 \nabla_{\mathbf{x}_1}\phi_{12}^2\cdot\frac{\partial f_1(\mathbf{X}_1)}{\partial \mathbf{v}_1} n(\mathbf{x}_2), \tag{8}$$

where $n(\mathbf{x}_2) = n_0\int \mathrm{d}\mathbf{v}_2 f_1(\mathbf{x}_2, \mathbf{v}_2)$ is the density distribution function and n_0 is the average equilibrium density. The third term of equation (8) gives the contribution to the kinetic equation due to pair correlations. While the average of ϕ_{12} is represented by a macroscopic potential $q\phi = \int \mathrm{d}X_2 f(X_2)\phi_{12}$, it is necessary to find a representation for the average of ϕ_{12}^2:

$$\phi_{12}^2 = \frac{1}{16\pi^2\epsilon_0^2}\frac{q^4}{|\mathbf{x}_1 - \mathbf{x}_2|^2}. \tag{9}$$

We want to calculate the k-space representation of ϕ_{12}^2:

$$\psi(k) = \frac{q^4}{(4\pi\epsilon_0)^2}\frac{1}{2\pi^3}\int \frac{1}{s^2}\exp(\mathrm{i}\mathbf{k}\cdot\mathbf{s})\,\mathrm{d}^3\mathbf{s} = \frac{1}{128\pi^5\epsilon_0^2}\frac{4\pi}{k}\int_R^\lambda \frac{\sin ks}{s}\,\mathrm{d}s, \tag{10}$$

where $s = |\mathbf{x}_1 - \mathbf{x}_2|$. The upper and lower limits in the integral appearing in equation (10) are taken to be λ and R instead of ∞ and 0. We can express $\psi(k)$ in terms of the exponential sine integral [23] as follows:

$$\psi(k) = \frac{1}{32\pi^4\epsilon_0^4 k}[s\mathrm{i}(k\lambda) - s\mathrm{i}(kR)]. \tag{11}$$

The upper limit λ is taken to be λ_D, since in a neutral plasma, polarization effects cause the potential to be screened on a distance corresponding to the Debye length. The lower limit cannot be taken to be zero because this will lead to a breakdown of the assumption that the expansion parameter g of BBGKY hierarchy should be small. Because of the strong interparticle interaction (large angle collisions) at close distances, it is not correct to assume $|g| \ll |f_1(1)f_1(2)|$. The smallness of g can be maintained only if the particles participating in the interaction be prevented to approach very close to each other. The short-range divergence that occurs in the context of the collision integrals of the Balescu–Lenard equation is resolved by simply cutting off the integral at some lower limit spatial scale. Since the divergence that occurs in such integrals is logarithmic and since it is such a slowly varying function of its argument, the 'classical distance of closest approach' or the Landau length (q^2/k_BT) would be a reasonable choice. In our particular problem, instead of logarithmic, the dependences of cutoffs are coming through sine integral function which is an even slower varying function than the logarithmic. Therefore, the concept of cutoff is also applicable in the present case and an appropriate value for the lower limit is taken to be the Landau length. Therefore, restricting the lower limit to $R = q^2/k_BT$ excludes the large-angle collisions in a plasma which is also consistent with the plasma approximation since the ratio of the cross sections for a single large-angle and multiple small-angle collisions falls off as $1/8ln(\Lambda)$ where $\Lambda = 24\pi n\lambda_D^3$. For a weakly non-ideal plasma with the g factor taken to be 0.1, the cross section for multiple small-angle collisions is about 50 times the cross section for a single large-angle collision showing that it is justified to neglect the latter while estimating the Fourier transform of the Coulomb potential. Such arguments also find relevance in the removal of logarithmic divergences that appear in the diffusion coefficients obtained from Fokker–Planck and Balescu–Lenard equations at small and large impact parameters for a Coulombian two-particle interaction.

We use the series expansion of the exponential sine integral to obtain $\psi(k)$ as the following:

$$= \sum_{n=0}^{\infty} \frac{(-1)^n[\lambda^{2n+1} - R^{2n+1}]}{(2n+1)(2n+1)!} k^{2n}. \tag{12}$$

The correlation term on the right-hand side of equation (8) works out as follows after considering the Coulomb potential for ϕ_{12}:

$$\mathbf{v}_1 \cdot \frac{\partial f_1}{\partial \mathbf{x}_1} - \frac{n_0}{m} \int \mathrm{d}\mathbf{X}_2 f_1(\mathbf{X}_2) \nabla_{\mathbf{x}_1} \phi_{12} \cdot \frac{\partial f_1(\mathbf{X}_1)}{\partial \mathbf{v}_1}$$
$$= -\frac{\omega_p^2}{32\pi^4 n_0^2 \lambda_D^2} \int \mathrm{d}\mathbf{x}_2 \nabla_{\mathbf{x}_1} \left(\frac{1}{|\mathbf{x}_1 - \mathbf{x}_2|^2} \right) \cdot \frac{\partial f_1(\mathbf{X}_1)}{\partial \mathbf{v}_1} n(\mathbf{x}_2). \tag{13}$$

Inserting the k-space representation of $1/|\mathbf{x}_1 - \mathbf{x}_2|^2$

$$\frac{1}{|\mathbf{x}_1 - \mathbf{x}_2|^2} = \int \mathrm{d}\mathbf{k} \frac{(si(k\lambda) - si(kR))}{k} \mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot(\mathbf{x}_1 - \mathbf{x}_2)}, \tag{14}$$

the first-order kinetic equation works out to be the following after the Fourier integrals are carried out:

$$\mathbf{v}_1 \cdot \frac{\partial f_1}{\partial \mathbf{x}_1} - \frac{n_0}{m} \int \mathrm{d}\mathbf{X}_2 f_1(\mathbf{X}_2) \nabla_{\mathbf{x}_1} \phi_{12} \cdot \frac{\partial f_1(\mathbf{X}_1)}{\partial \mathbf{v}_1}$$
$$= -\frac{\omega_p^2}{4\pi n_0^2 \lambda_D^2} \nabla_{\mathbf{x}_1} \int \mathrm{d}\mathbf{k} \left[\sum_{n=0}^{\infty} \frac{(-1)^n \left[\lambda_D^{2n+1} - R^{2n+1}\right]}{(2n+1)(2n+1)!} k^{2n} \right] n(\mathbf{k})\, \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}_1} \cdot \frac{\partial f_1(\mathbf{X}_1)}{\partial \mathbf{v}_1}. \tag{15}$$

Equation (16) gives a general representation of the term involving pair correlations as a series expansion involving the plasma density. It should be noted that the terms in equation (16) vanish for a homogeneous plasma. The first-order kinetic equation (normalized) for electrons(ions) considering both electron–electron(ion–ion) and electron–ion(ion–electron) correlations and retaining all the terms of the exponential sine integral expansion is given by the following:

$$\mathbf{v}_1 \cdot \frac{\partial f_1(\mathbf{X}_1)}{\partial \mathbf{x}_1} - \frac{q}{|q|} \frac{\partial \phi(\mathbf{x}_1)}{\partial \mathbf{x}_1} \cdot \frac{\partial f_1(\mathbf{X}_1)}{\partial \mathbf{v}_1}$$
$$= -A \nabla_{\mathbf{x}_1} \left[\sum_{n=0}^{\infty} \frac{[1 - (R/\lambda_D)^{2n+1}]}{(2n+1)(2n+1)!} \nabla^{2n}(n_i + n_e) \right] \cdot \frac{\partial f_1(\mathbf{X}_1)}{\partial \mathbf{v}_1}, \tag{16}$$

where $A = 1/3N_D$ and the variables x, n, v and ϕ are normalized by λ_D, n_0, $v_{\rm th}$ and $k_B T/|q|$ respectively, where λ_D, n_0, T and $v_{\rm th}$ are the Debye length, density, temperature and thermal velocity of the species. The parameter N_D denotes the number of particles in the Debye sphere. In normalized variables, identical equations are obtained for both electron and ions.

The above equation is utilized to obtain the equilibrium density distribution for electrons and ions in the presence of weak non-ideal effects. From the single-particle equation of motion

$$m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\frac{\partial}{\partial \mathbf{x}} \left[-\frac{q}{|q|}\phi + A \left(\sum_{n=0}^{\infty} \frac{[1 - (R/\lambda_D)^{2n+1}]}{(2n+1)(2n+1)!} \nabla^{2n}(n_i + n_e) \right) \right] \tag{17}$$

we construct the constant of motion that is utilized to obtain the following equilibrium distribution functions for electrons and ions:

$$n_e = \exp\left(\phi(r) + \frac{1}{3N_D} \left[\sum_{n=0}^{\infty} \frac{[1 - (1/N_D)^{2n+1}]}{(2n+1)(2n+1)!} \nabla^{2n}(n_i + n_e) \right] \right), \tag{18}$$

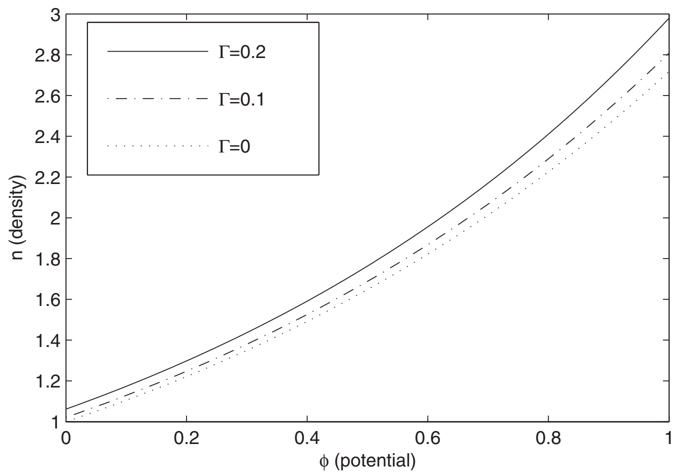

Figure 1. Normalized electron density n_e plotted against ϕ for $\Gamma = 0, 0.1, 0.2$.

$$n_i = \exp\left(-\phi(r) + \frac{1}{3N_D}\left[\sum_{n=0}^{\infty}\frac{[1-(1/N_D)^{2n+1}]}{(2n+1)(2n+1)!}\nabla^{2n}(n_i+n_e)\right]\right). \tag{19}$$

The role of weak non-ideal effects is displayed [24] through a modification of the equilibrium distribution function. In the limit $1/N_D \to 0$, the usual Boltzmann distribution is recovered.

3. Results and conclusions

In figure 1, we have plotted the electron density versus normalized potential for three different values of $\Gamma = 0$, 0.1 and 0.2. It may be noted that $g = 1/N_D = \Gamma^{3/2}$ so that the chosen values of Γ are within the limits imposed by the pertubation technique employed to obtain the density distribution. The curve corresponding to $\Gamma = 0.0$ refers to the usual Boltzmann distribution for an ideal plasma. The density profiles for $\Gamma = 0.1$ and 0.2 corresponding to a weakly non-ideal plasma show the influence of correlations on the density distribution.

The influence of discrete particle effects on the macroscopic phenomena in a plasma is studied by developing an equilibrium kinetic equation that contains first-order corrections in the plasma parameter. For a non-uniform plasma in thermal equilibrium, the Vlasov equation describes the density distribution in terms of the average electrostatic potential without any reference to the nature of the two-particle interaction in the plasma. This is because in the limit $g \to 0$ the BBGKY hierarchy equation takes care of collective effects only.

The first-order kinetic equation explicitly includes the binary interaction effects that are considered to be Coulombian in this work. In a manner similar to that utilized to develop the Vlasov equation, an attempt is made to express the first-order terms in terms of an average plasma potential. Interestingly, the nature of the two-particle interaction is also revealed through the first-order equation in the way the correlation term depends on the macroscopic potential or density. The entire scheme is facilitated by the observation of certain divergences that arise due to the nature of the Coulomb potential. A lower limit to the interparticle distance other than zero becomes relevant, in the absence of which the Mayer series expansion based on the smallness of g that leads to the truncation of BBGKY hierarchy loses its appropriateness.

Inclusion of the lower limit of impact parameter brings a significant alteration in the Fourier transform of the Coulomb potential. The last term in the Vlasov equation that has its root in the collective nature of the plasma system, does not carry the signature of the singular character of the Coulomb potential. The encounters at short distances give rise to large deviation collisions, and in a weakly coupled plasma their contribution is negligible with respect to the cumulative effects of the small angle collisions. Therefore, the lower limit of the interparticle distance is taken to be the Landau length (e^2/k_BT) that marks the boundary between large- and small-angle collisions. The upper limit to which interaction between particles can extend is the Debye length in view of the screening effects in a plasma. The choice of finite upper and lower limits enables a representation of the pair correlation terms in the kinetic equation through suitable operators acting on macroscopic variables. It is interesting to observe that the quantity arising on account of finite pair correlations depends on the exact form of binary interaction between particles. In the case of dusty plasmas, the interaction between two dust particles is known to be of the Debye–Huckel type [25]. However, for such and any other general form of binary interaction, it is not possible to find an operator that will make it possible to express the microscopic interaction term in terms of macroscopic variables. The nature of the Coulomb interaction between particles seems to afford a unique advantage in this respect.

It is important to understand the parameter region in the plasma where such weakly non-ideal effects will be important. For plasmas of very low density, the study of plasma phenomena in terms of only collective effects is justified. This situation leads to the small values of the order parameter so that the BBGKY hierarchy reduces to the Vlasov equation. For higher plasma densities, single-particle effects predominate and the Mayer cluster expansion loses its appropriateness in truncating the BBGKY hierarchy. Such plasmas are in the strongly coupled regime and their dynamics can be studied by several methods [26–28]. In the intermediate density region, single-particle effects described by binary interactions influence collective phenomena and for small but finite values of $g = 1/N_D$, such effects can be studied within the framework of the BBGKY hierarchy. The influence of weak non-ideal effects is to modify the equilibrium density distribution. Poisson's equation together with the modified density distribution will give rise to potential structures that depend on the correlation parameter. These solutions of the new equilibrium kinetic equation obtained by taking binary correlations into account correspond to the potential structures that are stationary nonlinear solutions of Vlasov–Poisson's equation. While correlations are known to be responsible to drive a system to equilibrium, under stationary conditions they act to modify the equilibrium density distribution.

References

[1] Gerard F 2002 *Advances in Plasma Physics Research* vol 4 (New York: Nova Science)
[2] Landau L D 1946 *J. Phys.* (Moscow) **10** 25
[3] Balescu R 1960 *Phys. Fluids* **3** 52
Lenard A 1960 *Ann. Phys.* **3** 390
[4] Lenard A and Bernstein I B 1958 *Phys. Rev.* **112** 1456
[5] Zakharov V E and Karpman V I 1963 *Sov. Phys.—JETP* **16** 351
[6] Brodin G 1997 *Phys. Rev. Lett.* **78** 1263
[7] Danielson J R, Anderegg F and Driscoll C F 2004 *Phys. Rev. Lett.* **92** 245003
[8] Valentini F 2008 *Phys. Plasmas* **15** 022102
Valentini F and Veltri P 2008 *Europhys. Lett.* **81** 15002
[9] O'Neil T and Rostoker N 1965 *Phys. Fluids* **8** 1109
[10] Carbone V, De Marco R, Valentini F and Veltri P 2007 *Europhys. Lett.* **78** 65001
[11] Gorman D and Montgomery D 1963 *Phys. Rev.* **131** 7

[12] Chavanis P H 2006 *Physica* A **361** 55
Chavanis P H 2006 *Physica* A **361** 81
[13] Bernstein I B, Greene J M and Kruskal M D 1957 *Phys. Rev.* **108** 546
[14] Bogoliubov N N 1965 *The Dynamical Theory in Statistical Physics* (Delhi: Hindustan)
[15] Montgomery D C and Tidman D A 1964 *Plasma Kinetic Theory* (New York: McGraw-Hill)
[16] Brown L S and Yaffe L G 2001 *Phys. Rep.* **340** 1–340
[17] Brouwer H H and Schram P P J M 1987 *Physica* A **141** 589
Brouwer H H and Schram P P J M 1988 *Physica* A **147** 567
Frieman E A and Book D L 1963 *Phys. Fluids* **6** 1700
[18] Bose A and Janaki M S 2007 *Phys. Plasmas* **14** 063701
[19] Murillo M S 2004 *Phys. Plasmas* **11** 2964
[20] Wang X and Bhattacharjee A 1998 *Phys. Rev.* E **58** 4967
[21] Deutsch C, Furutani Y and Gombert M M 1981 *Phys. Rep.* **69** 85
[22] Fortov V E, Gavrikov A V, Petrov O F, Shakhova I A and Vorobev V S 2007 *Phys. Plasmas* **14** 040705
Dubin D H E 1996 *Phys. Rev.* E **53** 5268
[23] Abramowitz M and Stegun I 1970 *Handbook of Mathematical Functions* (New York: Dover)
[24] Dwivedi C B and Bannur V M 1996 *Phys. Plasmas* **3** 685
[25] Wang X and Bhattacharjee A 1996 *Phys. Plasmas* **4** 1189
[26] Carini P, Kalman G and Golden K I 1982 *Phys. Rev.* A **26** 1686
[27] Ichimaru S, Iyetomo H and Tanaka S 1987 *Phys. Rep.* **149** 91
[28] Kaw P K and Sen A 1998 *Phys. Plasmas* **5** 3552